The Latitudes of the Northern and Southern Bases of the Great Pyramid and its Correlations to the Lengths of the Solar (Tropical) and Sidereal Years

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Abstract

The author provides easily verifiable evidence that the ancient Egyptians must have used the same system of latitudes we use today, had knowledge of the exact lengths of the solar (tropical) and sidereal years (to the fourth decimal place), and incorporated these values into their placement of the Great Pyramid. This predates historical accounts of the invention of the geographic coordinate system (circa 220-150 BCE) and the discovery of the distinction between the solar and sidereal years (circa 130 BCE) by at least 2300 years, given the date of construction of the Great Pyramid (circa 2550 BCE).

Keywords: geographic coordinate system, system of latitudes, geodesy, mathematics, astronomy, length of solar year, length of sidereal year, ancient Egypt, the Great Pyramid.

1.Introduction

The ancient Greeks are usually credited as being the first to have conceived the very idea of the geographic coordinate system we use today (with parallels of latitude and meridians of longitude dividing the circle in 360 degrees),[1] as well as being the first to define precession[2] and the distinction between the solar[3] and sidereal[4] years.[5] While there are no known written records in which the ancient Egyptians expressed knowledge of precession or the use of a coordinate system, I propose that the geographic placement of the Great Pyramid indicates that the ancient Egyptians must have used the same system of latitudes we use today and must have had knowledge of the exact lengths of the solar and sidereal years (to the fourth decimal place).

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[1] Booth (1995), p.239; Kerski (2016), p.167; Boyer and Merzbach (2011), p.147; Sidoli (2004), pp. 71-84.

[2] Precession, responsible for the distinction between the solar and sidereal years, is a change in the orientation of the Earth's rotational axis in a cycle of approximately 26,000 years.

[3] Solar year, also known as tropical year, is the length of time it takes for the earth to complete one revolution around the sun (365.2421 days), from vernal equinox sunrise to vernal equinox sunrise for example.

[4] Sidereal year is the length of time it takes for the earth to complete one orbit around the sun with respect to the stars. In other words, the sidereal year measures the length of time between star rises (365.2563 days) while the solar year measures the length of time between sunrises (365.2421 days).

[5] Lewis (1862), p. 213; Needham (1959), p. 270; Toomer (1984), p. 81; Duke (July 2002) pp. 427-433.

2. Discussion

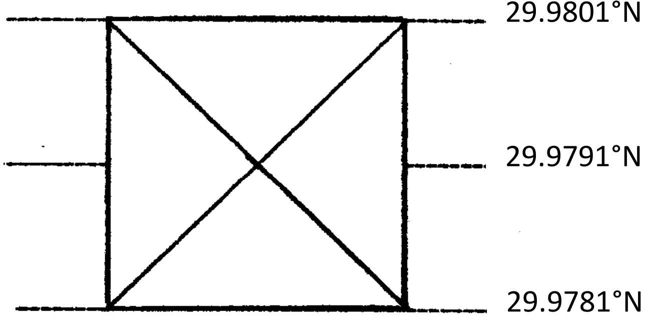
The apex of the Great Pyramid is 29.9791 degrees north of the equator (29.9791°N).[6] It has been proposed that the ancient Egyptians had intended to place the Great Pyramid at the parallel 30 degrees north of the equator (i.e. one third of the way between the equator and the North Pole, or one twelfth of a circle) but missed for various reasons, including due to atmospheric refraction.[7] If the intention of the builders of the Pyramid was to place it at the 30 degrees north parallel, then they missed by 0.0209 degrees (a distance of more than twenty three hundred meters). Instead of missing by 0.0209 degrees, I propose that the builders of the Great Pyramid placed it to an accuracy to the fourth decimal place (accurate within ten meters), and in a more complex mathematical relationship to the parallel 30. In Figure 1, the Great Pyramid is shown with the parallels of latitude that cross its northern base, its apex, and its southern base. The values of these latitudes are shown to the right of the pyramid. To the left, three sets of equations show that these latitudes are in mathematical relationship to the values[8] of the solar, average, and sidereal years, vis-à-vis the calendar year and the parallel 30.[9]

Figure 1
The placement of the Great Pyramid incorporates knowledge of the solar and sidereal years

$$\frac{30^\circ}{\substack{Solar\ year \\ (365.2421\ days)}} = \frac{29.9801^\circ}{\substack{Calendar\ year \\ (365\ days)}}$$

$$\frac{30^\circ}{\substack{Av.\ length\ of\ year \\ (365.2500\ days)}} = \frac{29.9791^\circ}{\substack{Calendar\ year \\ (365\ days}}$$

$$\frac{30^\circ}{\substack{Sidereal\ year \\ (365.2563\ days)}} = \frac{29.9781^\circ}{\substack{Calendar\ year \\ (365\ days)}}$$



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[6] Coordinates source: Kingsland (1972) p. 3; LatLong.net; GeoHack at Wikipedia; and Google Earth.

[7] Lancaster-Brown (2013), p. 270-271; Thurston (December 2003), pp. 6; Proctor (1879), p. 264; Proctor (1883), p. 106, 83-84; Proctor (1893), p. 57-59; Smith (1867), p.46-47; Hancock and Bauval, (Dec 13, 1993) p. 49-50.

[8] Doggett (NASA website) Section 1.1; Meeus and Savoie (1992), pp. 40; Borkowski, (June 1991), pp. 121-130; O'Neil (1976), p. 22.

[9] In the first equation, the ratio of the fraction noted as 30 degrees over the solar year (30°/365.2421 days) is, to the fifth decimal place, equal to the ratio of the fraction noted as 29.9801 degrees over the calendar year (29.9801°/365 days).
In the next equation, the ratio of the fraction noted as 30 degrees over the average length of the year (30°/365.2500 days) is, to the fifth decimal place, equal to the ratio of the fraction noted as 29.9791 degrees over the calendar year (29.9791°/365 days).
In the last equation, the ratio of the fraction noted as 30 degrees over the sidereal year (30°/365.2563 days) is, to the fifth decimal place, equal to the ratio of the fraction noted as 29.9781 degrees over the calendar year (29.9781°/365 days).
All six fractions share the same ratio (0.08213) to the fifth decimal place.

The mathematical evidence illustrated in Figure 1 indicates that the builders of the Great Pyramid: (1) had knowledge of the exact lengths of the solar and sidereal years to the fourth decimal place; (2) used the same system of latitudes we use today, from zero degrees at the equator to 90 degrees at the poles (a total of 360 degrees); and (3) planned the placement, orientation, and dimensions of the base of the Great Pyramid according to the aforementioned equations.

The Greek astronomer, geographer, and mathematician Hipparchus of Nicea is usually credited as being the first to define precession (circa 130 BCE), and the distinction between the solar and sidereal years.[10] However, the geographic placement of the Great Pyramid, shown in Figure 1, indicates that the ancient Egyptians had this knowledge at least 2400 years before Hipparchus.

The equations presented in Figure 1 reveal that the builders of the Great Pyramid used the same system of degrees of latitude we use today. There are no known records that indicate or even suggest that the ancient Egyptians invented this system of degrees of latitude, much less that they employed it in such a sophisticated fashion. The ancient Greeks are usually credited with the invention of the geographic coordinate system we use today (circa 220-150 BCE).[11] The mathematical evidence in Figure 1, however, reveals that the builders of the Great Pyramid must have established and employed the same system of latitudes millennia before the birth of ancient Greek civilization.

This system of latitudes could have been developed independently more than once because it is based upon natural principles of geodesy, astronomy, geometry, and mathematics. It measures the angular distance of a place north or south of the equator, starting at zero degrees at the equator to 90 degrees at the poles (a total of 360 degrees). The equator provides a natural starting position for measuring latitude (it is a great circle, perpendicular to the axis of the earth, and equidistant everywhere from the poles). Several ancient peoples divided the circle into 360 degrees (or parts), or used a calendar consisting of 12 months of 30 days each (a total of 360 days) including the ancient Egyptians.[12] The choice of measuring the circumference of a circle, and the length of a year, in 360 units

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[10] Lewis (1862), p.213; Needham (1959), p.270; Toomer (1996), p.81.

[11] Sickle (2004), p.2; Hansen and Gray (2010), p. 43; Kerski (2016), p. 167.

[12] For example: (1) the ancient Egyptian civil calendar was established around 4200 BCE and it consisted of 12 months of 30 days each for a total of 360 days. In between the end of the old year and the beginning of the new year they celebrated 5 epagomenal days, viewed as a transitional period separate from the old and the new year; (2) the Rigveda (1700-1100 BCE) contains a passage that describes a wheel with 12 spokes and 360 pegs; (3) the ancient Greeks first divided the circle into 360 parts around the second century BCE, but the system is presumed to have originated in Babylonian astronomy; (4) the Babylonian calendar dates back to around 2000 BCE and it also consisted of 12 months of 30 days each for a total of 360 days; and (5) from around the fifth century BCE the wheel of the zodiac was divided into 12 constellations each containing a space of exactly 30 degrees of celestial longitude, even though there are at least 13 constellations in the plane of the ecliptic and their celestial longitude range from around 10 degrees (Scorpius) up to around 45 degrees (Virgo).

References for footnote 11: (1) Breasted (1914), p. 23-24; West (1993), p. 95, 99; Neugebauer (1969), p. 25, 81; (2) Griffith (1896), Hymn 1.164.48, p. 90; Kak (2003), p.863; Dietrich (2011), p.164; (3) Hansen and Gray (2010), p. 43; Neugebauer (1969), p. 25; Linton (2004), p. 52;

further divided into 12 parts each containing 30 units could have occurred independently more than once because these values are rooted in astronomy and mathematics.[13] The number 360 is a close approximation to the number of days in a year (365.25 days on average), as well as to the number of days in 12 full moons (354.36 days), and it is an excellent number for an efficient mathematical system.[14]

It is possible that we have indirectly inherited these systems from the ancient Egyptians through ancient Greece. We know that there was contact between Greece and Egypt, and in 332 BCE Alexander the Great conquered Egypt for the Macedonian Greek Empire. There are, however, no known records suggesting the ancient Greeks adopted this system of latitudes from Egypt. Again, given the fact that this system of latitudes is rooted on geodesy, astronomy, and mathematics it could have emerged independently more than once.

3.Conclusion

The Great Pyramid is the only pyramid placed at this set of latitudes, and therefore it is the only pyramid that presents the correlations displayed in Figure 1. It seems unlikely that the latitudes of the Great Pyramid present these correlations by chance. Ancient Egypt was one of the most advanced civilizations of its time: they invented a written language; established a centralized government and laws; built cities with streets, libraries, restaurants, markets, and religious centers; built the first known ships; and developed several disciplines including medicine, agriculture, astronomy, mathematics, and engineering.[15] The Great Pyramid is arguably the greatest engineering achievement of ancient Egypt.[16] It is the largest megalithic monument ever built, and for thousands of years it was the tallest building in

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Haselberger (2000), p. 284, 288; Conway and Guy (2012), p. 17; (4) Cajori (1893), p. 7, 8; Neugebauer (1969), p. 25, 81; Englund (1988), p.124-125; (5) McClure (2015); Plait (2016).

[13] Dunham (1994), p. 184; Aufmann and Lockwood (2012), p. 40; Narrien (1883), p. 85; Kak (2003), pp. 847, 858, 863.

[14] For example: (1) the number 360 is a **highly composite number** with 24 divisors and the smallest number divisible by every **natural number** except 7; consequently, (2) in a system of 360 degrees a right angle has 90 degrees (360°÷4=90°), an equilateral triangle has three angles of 60 degrees each (180°÷3=60°), and one twelfth of a circle is exactly 30 degrees (360°÷12=30); (3) if, instead of dividing the circle in 360 degrees, we were to divide the circle according to the average length of the year (365.25), a right angle would have 91.3125 degrees (365.25°÷4=91.3125°), an equilateral triangle would have three angles of 60.875 degrees each (365.25°÷3=60.875°), and one twelfth of a circle would have the value of 30.4375 degrees (365.25°÷12=30.4375°); and (4) if we were to divide the circle according to the length of the year, which length of the year should be used? There are several candidates: the calendar year (365 days); the calendar leap year (366 days); the solar year (365.242 days); the sidereal year (365.256 days); the average length of the year (365.250 days); the length of a lunar calendar (354 days); and the length of 12 full moons (354.36 days); (5) in addition, the average between the leap year calendar with 366 days and the lunar calendar with 354 days is exactly 360 days ((366+354) ÷ 2 = 360). In conclusion, the use of a calendar with 360 days and the division of the circle into 360 degrees are rooted in astronomy and mathematics.

[15] Lubicz (1982); Lloyd (2014); Ward (2000); Allen (2005); Allen et al. (1999); Riggs (2014); Clagett (1999); Clagett (1995); Lucas and Harris (1999); Magli (2013); Rossi (2004); Lubicz (1977).

[16] Messler (2012), p.8, 11; Romer (2007), p.24, 81, 17; Kingsland (1972), p.1-2; Neugebauer (1950) pp. 1.

the world.[17] The Great Pyramid is also one of the most symmetrical structures in the world, and it is well known that the monument was laid out and oriented according to geometrical and astronomical principles.[18] The base of the Great Pyramid, for example, is nearly a perfect square and its four sides are oriented towards the cardinal points with a precision that could only have been achieved through prolonged and careful astronomical observation.[19] Given both the level of astronomical knowledge possessed by ancient Egypt and the extraordinary level of planning and deliberation that must have gone into the monument's design, the mathematical correlation between the latitudes of the northern and southern bases of the Great Pyramid to the solar and sidereal years discussed in this paper (see Figure 1) is unlikely to be coincidental.

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[17] Romer (2007), p. 13, 425; Newbolt (2014), chapter 1; Glenday (2010), p. 334; Cawley (2016), p. 1-2.

[18] Cole (1925), pp. 1-11; Romer (2007), p. 42, 346-347, 353-356, 58-66, 2; Herz-Fischler (2000); Neugebauer (1950), pp. 1-3; Neugebauer (1969), p. 96; Thurston (December 2003), pp. 4-11; Thurston (1922), p. 25; Messler (2013), p. 29-35

[19] Romer (2007), p. 13, 42, 346-347, 61, 197, 4; Kingsland (1972), p. 3; Belmonte (2001), pp. S1; Thurston (1922), p. 25; Otto Neugebauer (1950), pp. 1-3; Neugebauer (1969), p. 96; Livio (2002), p. 52; Cole (1925), pp. 6, 10, 11

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